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Gauge unification in supersymmetric intersecting brane worlds

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Abstract: We show that contrary to first expectations realistic three generation supersymmetric intersecting brane world models give rise to phenomenologically interesting predictions about gauge coupling unification. Assuming the most economical way of realizing the matter content of the MSSM via intersecting branes we obtain a model independent relation among the three gauge coupling constants at the string scale. In order to correctly reproduce the experimentally known values of $\sin^2\theta_w(M_z)$ and $\alpha_s(M_z)$ this relation leads to natural gauge coupling unification at a string scale close to the standard GUT scale 2×10^{16} GeV. Additional vector-like matter can push the unification scale up to the Planck scale.

Keywords: Superstrings and Heterotic Strings, D-branes, String Duality, Superstring Vacua.

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1. Introduction

The experimental observation [1] that all three momentum dependent gauge coupling constants of the minimal supersymmetric Standard Model (MSSM), although being rather different at the low energy scale M_Z, apparently meet at a high mass scale M_X ($M_X \gg M_Z$) is very striking and naturally calls for a convincing theoretical explanation, commonly called *Grand Unification*. This means that above M_X all interactions of the Standard Model should be unified into a single theory, which predicts the observed relations among the coupling constants in a natural manner. At the one-loop level, the gauge couplings of the low energy Standard Model gauge theory evolve as

$$\frac{4\pi}{g_a^2(\mu)} = k_a \frac{4\pi}{g_X^2} + \frac{b_a}{2\pi} \log\left(\frac{\mu}{M_X}\right) + \Delta_a \qquad (1.1)$$

with $a = \{\mathrm{SU}(3), \mathrm{SU}(2), \mathrm{U}(1)_Y\}$. Here, the b_a are the renormalization group coefficients which can be computed by knowing the charges of the particle spectrum with respect to the three gauge groups. The k_a are constants which parameterize the tree level relations among the gauge couplings at the unification scale M_X, and which should be predicted completely or at least in part by the GUT theory. Finally, the quantities Δ_a are the one-loop group dependent threshold corrections due to heavy states at the scale M_X. They should also be calculable in any given GUT theory. Therefore without further input these three equations contain as parameters the three experimentally known couplings $g_a(M_Z)$ and the a priori unknown parameters M_X and g_X.

Starting from the measured low-energy data, namely from $\alpha_3(M_Z)$, $\alpha(M_Z)$ and $\sin^2\theta_w(M_Z)$, and extrapolating to high energies by the use of the spectrum of the MSSM in the renormalization group equations (1.1), it turns out that at a scale $M_X \simeq 2\cdot 10^{16}$ GeV the constants k_a obey $k_3 \simeq k_2 \simeq \frac{3}{5}k_1$ to a surprising level of accuracy. Therefore this scale suggests itself as a natural unification scale. However it us useful to emphasize that different unification scales are needed if the GUT theory provides different relations among the k_a.

In supersymmetric field theory, GUT theories [2] with unifying gauge group $G \supset \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y$ provide a beautiful theoretical framework. In the case of $G = \mathrm{SU}(5)$ there are indeed two fixed relations among the gauge couplings at M_X provided by the group structure of SU(5), namely $k_1 = 5/3$, $k_2 = k_3 = 1$. This implies that with $\alpha_3(M_Z) \simeq 0.12$ and $\alpha(M_Z) \simeq 1/128$, and assuming that the one-loop threshold corrections are small in field theory, two renormalization group equations fix $M_X \simeq 2\cdot 10^{16}\,\mathrm{GeV}$ and $\alpha_X \simeq 1/24$; then the remaining equation provides a prediction for the coupling $\sin^2\theta_w(M_Z)$. Using the spectrum of the MSSM, the SU(5) GUT theory so far is in excellent agreement with the experimental measurements. This is depicted in figure 1 (the upper curve denotes the running of $\frac{3}{5}\alpha_Y^{-1}$).

String theoretical *Grand Unification* is more ambitious than field theory, since it attempts to unify gravity with the gauge interactions. Therefore one expects that in string theory unification takes place at the string scale M_s, i.e. $M_X = M_s$. This scale should be related to the Planck mass $M_{pl} = G_N^{-1/2} \simeq 1.2\cdot 10^{19}\,\mathrm{GeV}$ in one way or the other. Moreover at M_s all gauge couplings should be expressible in terms of the string coupling constant g_{st}, making the constants k_a in principle calculable. Perturbative heterotic string compactifications[1] provide a very concrete realization of string gauge coupling unification. Here the constants k_a are just the Kac-Moody levels of the corresponding gauge Kac-Moody algebras [4], where the most favorite (SU(5) like) choice is $k_3 = k_2 = \frac{3}{5}k_1 = 1$ (however see also the discussion in [5]). Furthermore, the heterotic string scale M_s can be related to the Planck mass as follows [6, 7]

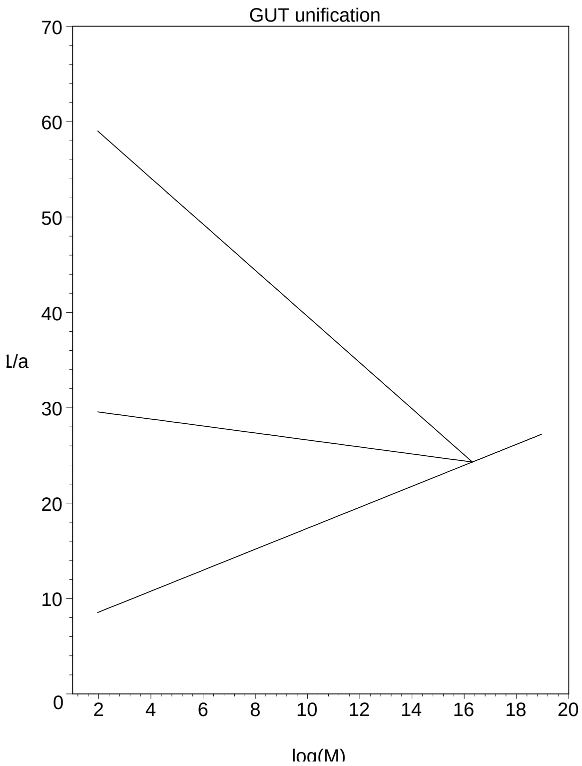

Figure 1: GUT unification.

$$M_s \simeq g_{st}\cdot 0.058\cdot M_{pl}\,. \tag{1.2}$$

This relation eliminates one parameter from the three eqs. (1.1), so knowing $\alpha(M_Z)$ one can predict both $\alpha_3(M_Z)$ and $\sin^2\theta_w(M_Z)$. Assuming the spectrum of the MSSM and neglecting one-loop threshold corrections the heterotic string prediction is off the experimental values for these two couplings by several standard deviations. In other words, with $g_{st} \simeq 0.7$ the heterotic string scale becomes too large, namely $M_s \simeq 5\cdot 10^{17}\,\mathrm{GeV}$. As being roughly of the same size as the unification scale of $2\times 10^{16}\,\mathrm{GeV}$, where the three coupling constants of the MSSM meet according to their experimental values, this was

[1]See [3] for a review.

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considered to be an encouraging result for heterotic string gauge unification. Nevertheless, the remaining discrepancy between the unification scale and M_s is too large in order to be neglected. Therefore the influence of extra states at intermediate mass scales [8] was discussed in order to improve the situation.

Alternatively the effect of one-loop string threshold corrections Δ_a [6, 9, 10] due to heavy string modes was investigated. The quantities Δ_a depend in general on the gauge group and induce a deviation of the relations among the gauge couplings at M_s from their tree level values (absorbing this effect into the k_a, one gets sort of effective one-loop Kac-Moody levels k_a^{1-loop}). In addition, in [11] it was shown that in heterotic M-theory compactifications the string scale could be lowered such that agreement with the experiment is in principle possible. However the predictive power of the heterotic string gauge unification is somehow weakened by all these considerations.

More recently, gauge coupling unification was also discussed in the framework of non-supersymmetric large extra dimension scenarios with a string scale in the TeV region. In one proposal one uses polynomial running of the gauge couplings [12], whereas the second approach outlined in [13] uses ideas similar in spirit to what we are to present in the following (see also [14]).

In this paper we want to investigate the question in how far intersecting brane world (IBW) models [15]–[35] can predict the values of the low-energy gauge couplings in a satisfactory way. The main new ingredient in these models is that they contain intersecting D-branes and open strings in a consistent manner providing simple mechanisms to generate non-abelian gauge interactions and chiral fermions in a systematic way. To be specific we will consider type IIA orientifolds with several stacks of $D6_a$-branes, each being wrapped around individual compact homology 3-cycles π_a of the internal space. Hence the effective open string gauge theories with groups G_a live in the 7-dimensional subspaces $\mathbb{R}^{1,3} \otimes \pi_a$. The chiral matter fields are located at the intersection points of the D6-branes, which are points on the internal space, i.e. the chiral matter fields live just in $\mathbb{R}^{1,3}$. In this way one can systematically build intersecting brane world models which come remarkably close to the (supersymmetric) Standard Model.

In contrast to the heterotic string, here each gauge factor comes with its own gauge coupling, which at string tree-level can be deduced from the Dirac-Born Infeld action. Here we simply state the well known result that the tree level gauge coupling at the string scale is given by[2]

$$\frac{4\pi}{g_a^2} = \frac{M_s^3\, V_a}{(2\pi)^3\, g_{st}\, \kappa_a}\,, \tag{1.3}$$

where V_a denotes the volume of the 3-cycle the $D6$-branes are wrapped on. The extra factor κ_a is related to the difference between the gauge couplings for $\mathrm{U}(N_a)$ and $\mathrm{SP}(2N_a)/\,\mathrm{SO}(2N_a)$ branes, namely $\kappa_a = 1$ for $\mathrm{U}(N_a)$ and $\kappa_a = 2$ for $\mathrm{SP}(2N_a)/\,\mathrm{SO}(2N_a)$. Note, that in deriving (1.3) one has normalized the gauge fields in the canonical way, i.e. $A_\mu = A_\mu^a\, T_a$ with $\mathrm{tr}(T_a\, T_b) = 1/2$. We see that by setting $g_{st} = g_X$ the constants k_a in

[2] A very explicite derivation is for instance contained in [35].

eq. (1.1) can by identified as

$$k_a = \frac{M_s^3\, V_a}{(2\pi)^3\, \kappa_a}\,. \tag{1.4}$$

The constants k_a now depend on the internal volumes V_a and since they are in general different, the k_a are generically independent and different. Hence gauge coupling unification does not seem to occur in type II brane world models in a very natural way.

By dimensionally reducing the type IIA gravitational action one can similarly express the Planck mass in terms of stringy parameters

$$M_{pl}^2 = \frac{8\, M_s^8\, V_6}{(2\pi)^6\, g_{st}^2}\,, \tag{1.5}$$

where V_6 is the overall volume of the Calabi-Yau manifold. The two equations (1.3) and (1.5) can be used to eliminate the unknown string coupling constant

$$\frac{1}{\alpha_a} = \frac{M_{pl}}{2\sqrt{2}\,\kappa_a\, M_s}\frac{V_a}{\sqrt{V_6}}\,. \tag{1.6}$$

Let us point out, that in that form the gauge coupling α_a depends on the complex structure moduli only. On the other hand the size of the Planck mass (1.5) is governed by the overall volume V_6.

Since, as already emphasized, each gauge coupling depends on the size of the 3-cycle the $D6$ is wrapped on, one might think that all gauge couplings are in general independent and gauge unification does not give very restrictive constraints. In the following section we will show that in a model independent bottom up approach the three phenomenological requirements, namely of (i) that the standard model branes mutually preserve $\mathcal{N}=1$ supersymmetry, of (ii) realizing a 3-generation MSSM like model from intersecting branes, i.e. realizing the intersection numbers of a 3 generation MSSM, and of (iii) getting a massless $U(1)_Y$ gauge boson with the correct hypercharges of the matter fields, lead in a natural way to one non-trivial relation among the tree level gauge couplings. Against our first intuition this relation allows for natural gauge coupling unification and makes a prediction about the string scale in this class of supersymmetric intersecting brane world models.

2. Realizing MSSM like models in intersecting brane worlds

The idea of intersecting brane worlds can be described very simple. One starts with an orientifold of Type IIA string theory on a Calabi-Yau space. The anti-holomorphic orientifold projection introduces $O6$ planes in the background, which are wrapped on some of the 3-cycles π_{O6} of the Calabi-Yau. These orientifold planes are charged under the Type IIA R-R 7-form and due to its tension also couple to the gravitational field. Since on a compact background the sum of all charges must vanish, one has to introduce $D6$-branes into the background as well, so that the overall 7-form charge vanishes. In general it is not necessary to place these $D6$-branes right on top of the orientifold planes, instead they can in general wrap different 3-cycles π_a of the underlying Calabi-Yau manifold. The condition

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that the overall 7-form charge vanishes gives rise to the following R-R tadpole cancellation condition,

$$\sum_a N_a\,(\pi_a + \pi'_a) - 4\,\pi_{O6} = 0\,, \tag{2.1}$$

where π'_a denotes the orientifold image of the cycle π_a.

In general such an arrangement will break supersymmetry, but non-trivial models have been explicitly constructed, where an $\mathcal{N}=1$ supersymmetry is preserved by all $D6$ branes. Moreover, it has been pointed out that at strong coupling such supersymmetric models do lift to M-theory compactifications on singular G_2-manifolds [36, 23].

Representation	Multiplicity
$[\mathbf{A_a}]_L$	$\frac{1}{2}\,(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a)$
$[\mathbf{S_a}]_L$	$\frac{1}{2}\,(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a)$
$[(\overline{\mathbf{N}}_\mathbf{a}, \mathbf{N_b})]_L$	$\pi_a \circ \pi_b$
$[(\mathbf{N_a}, \mathbf{N_b})]_L$	$\pi'_a \circ \pi_b$

Table 1: Chiral spectrum.

From the phenomenological point of view such string models are interesting, as they allow to search systematically for stringy realizations of the standard model respectively the MSSM. The gauge degrees of freedom are confined to the world volume of the $D6$-branes. In particular each stack of D6-branes wrapping a 3-cycle, which is not invariant under the orientifold projection gives rise to a gauge factor $\mathrm{U}(N_a)$. If however the 3-cycle is invariant one can also get gauge groups $\mathrm{SP}(2N_a)$ and $\mathrm{SO}(2N_a)$. The chiral matter fields are localized at the four-dimensional intersection locus between two $D6$-branes. The number of such intersections is given by the topological intersection number between the two different 3-cycles the $D6$-branes are wrapped on. Therefore, such models naturally give rise to chirality and family replication. The general chiral spectrum in terms of the topological intersection numbers is shown in table 1 [26].

So far concrete intersecting brane world models have been constructed on the torus T^6, on toroidal orbifolds and also on the quintic Calabi-Yau manifold [37]. In particular, supersymmetric standard-like models have been studied on the $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$ [23, 28, 34], $T^6/\mathbb{Z}_4$ [29] and $T^6/\mathbb{Z}_4\times\mathbb{Z}_2$ [31] orbifold. However, we think it is fair to say that none of these models are completely realistic yet. However, there are some general patterns supersymmetric standard-like models apparently share. In the following our bottom up analysis will not rely on any concrete model, but will only make use of these general properties which follow from the three phenomenological requirements spelled out at the end of the introduction.

First there are so far two simple ways to embed the standard model gauge group into products of unitary and symplectic gauge groups. Both of them use four stacks of $D6$-branes, which give rise to the initial gauge symmetries

$$\begin{aligned} A:&\ \ \mathrm{U}(3)\times\mathrm{SP}(2)\times\mathrm{U}(1)\times\mathrm{U}(1) \\ B:&\ \ \mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)\,. \end{aligned} \tag{2.2}$$

The difference is that in the first example [38] the $\mathrm{SU}(2)_L$ symmetry of the standard model is realized directly as an $\mathrm{SP}(2)=\mathrm{SU}(2)$ gauge group on the branes. In the second case

field	sector	I	$SU(3) \times SU(2) \times U(1)_a \times U(1)_c \times U(1)_d$
q_L	(ab)	3	$(3, 2; 1, 0, 0)$
u_R	(ac)	3	$(\overline{3}, 1; -1, 1, 0)$
d_R	(ac')	3	$(\overline{3}, 1; -1, -1, 0)$
e_L	(db)	3	$(1, 2; 0, 0, 1)$
e_R	(dc')	3	$(1, 1; 0, -1, -1)$
ν_R	(dc)	3	$(1, 1; 0, 1, -1)$

Table 2: Chiral spectrum for the A model.

one embeds $SU(2)_L$ into the U(2) gauge factor [19, 20]. Since the anomaly conditions are less constraining for the class A and the Standard Model embedding is more natural, let us explore this example in more detail.

Second, the chiral spectrum of the intersecting brane world model should be identical to the chiral spectrum of the standard model particles. This fixes uniquely the intersection numbers of the 3-cycles, $(\pi_a, \pi_b, \pi_c, \pi_d)$, the four stacks of $D6$-branes are wrapped on.

Note, that the $U(1) \subset U(N)$ gauge factors in table 2 are not canonically normalized. The canonically normalized ones are given by $\widetilde{U(1)} = U(1)/\sqrt{2\,N}$. The hypercharge Q_Y is given as the following linear combination of the three U(1)s

$$Q_Y = \frac{1}{3} Q_a - Q_c - Q_d\,. \tag{2.3}$$

As first described in [20] in general some of the stringy U(1)s are anomalous and get a mass via some generalized Green-Schwarz mechanism. However, for intersecting brane worlds it can also happen that via axionic couplings some anomaly-free abelian gauge groups become massive.

The condition that a linear combination $U(1)_Y = \sum_i c_i\, U(1)_i$ remains massless reads [26]

$$\sum_i c_i\, N_i\, \left(\pi_i - \pi'_i\right) = 0\,. \tag{2.4}$$

In general, if the hypercharge is such a linear combination of U(1)s, $Q_Y = \sum_i c_i Q_i$, then the gauge coupling is given by

$$\frac{1}{\alpha_Y} = \sum_i \frac{N_i\, c_i^2}{2} \frac{1}{\alpha_i}\,, \tag{2.5}$$

where we have taken into account that the U(1)s in table 2 are not canonically normalized. Therefore, in our case the gauge coupling of the hypercharge is given as

$$\frac{1}{\alpha_Y} = \frac{1}{6}\frac{1}{\alpha_a} + \frac{1}{2}\frac{1}{\alpha_c} + \frac{1}{2}\frac{1}{\alpha_d}\,. \tag{2.6}$$

Since naively one would guess that α_c and α_d are independent parameters one cannot derive any definite low-energy prediction from this formula. However, there exists a most natural and economical way of realizing the Standard Model intersection numbers.

Say one finds two supersymmetric 3-cycles π_a and π_b with the intersection numbers $\pi_a \circ \pi_b = 3$ then homologically choosing $\pi_d = \pi_a$ gives the right intersection numbers for π_d. Therefore the two volumes V_a and V_d have to agree: $V_a = V_d$. This also follows from

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the fact that the gauge symmetry can be enhanced $\mathrm{U}(3)_a \times \mathrm{U}(1)_d \to \mathrm{U}(4)$, if we put the brane d on top of the branes a. The condition (2.4) that $\mathrm{U}(1)_Y$ remains massless simply implies homologically $\pi_c' = \pi_c$, i.e. by really placing π_c on top of an orientifold plane one can enhance the gauge group for π_c from U(1) to SP(2). Therefore, at the bottom of this simple realization there lies an extended Pati-Salam like model[3] discussed $\mathrm{U}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2)$. On the one hand, from the field theory point of view it is very natural to assume, that the two gauge couplings of the two SU(2) factors have the same gauge coupling, i.e. there is an additional $\mathbb{Z}_2$ symmetry.[4] On the other hand, from the stringy point of view, even though we cannot rigorously prove it in the general case, the constraints from supersymmetry, i.e. the requirement that all branes are calibrated in the same way, and the intersection numbers $\pi_a \circ \pi_b = -\pi_a \circ \pi_c = 3$ do not seem to leave very much room to evade that the internal volumes for the cycles π_b and π_c agree: $V_b = V_c$. Following these arguments it follows that $\alpha_d = \alpha_a = \alpha_s$ and $\alpha_c = \frac{1}{2}\alpha_b = \frac{1}{2}\alpha_w$. In all known concrete supersymmetric models, like the one discussed in [38], these relations among the gauge couplings hold[5]

This natural choice leads to a non-trivial relation among the three Standard Model gauge couplings at the string scale

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}\,. \tag{2.9}$$

Note, that this relation is compatible with the SU(5) GUT prediction

$$\frac{1}{\alpha_s} = \frac{1}{\alpha_w} = \frac{3}{5}\frac{1}{\alpha_Y}\,, \tag{2.10}$$

[3] Gauge unification in Pati-Salam like D-brane models has been discussed in refs. [39].

[4] This $\mathbb{Z}_2$ symmetry could be understood in some decompactification limit where $\mathrm{SP}(2)_b \times \mathrm{SP}(2)_c$ might be unified to SP(4).

[5] In toroidal models with four stacks of $D6_a$ branes ($a = 1, \ldots, 4$) wrapped on non-trivial 3-cycles of a six-dimensional torus $T^6 = \prod_{j=1}^3 T^{2,j}$ with the three complex structures $U^j = R_2^j/R_1^j$ and wrapping numbers (n_a^j, m_a^j) one obtains for the gauge couplings for each stack a

$$\begin{aligned}\alpha_a^{-1} &= \frac{1}{2\sqrt{2}\kappa_a}\,\frac{M_{pl}}{M_s}\,\times \\ &\times \left(n_a^1 n_a^2 n_a^3 \frac{1}{\sqrt{U^1U^2U^3}} - n_a^1 m_a^2 m_a^3 \sqrt{\frac{U^2U^3}{U^1}} - m_a^1 n_a^2 m_a^3 \sqrt{\frac{U^1U^3}{U^2}} - m_a^1 m_a^2 n_a^3 \sqrt{\frac{U^1U^2}{U^3}}\right).\end{aligned} \tag{2.7}$$

in the case of a supersymmetric cycle, i.e. $\sum_{j=1}^3 \arctan\left(m_a^j R_2^j / n_a^j R_1^j\right) = 0$. A possible choice for the wrapping numbers that fulfills the conditions worked out before is [38]: $\pi_a = \{(1,0), (1/\rho, 3\rho), (1/\rho, -3\rho)\}$, $\pi_b = \{(0,1), (1,0), (0,-1)\}$, $\pi_c = \{(0,1), (0,-1), (1,0)\}$, $\pi_d = \pi_a$, with $N_a = 3$, $N_b = N_c = N_d = 1$. $\mathcal{N} = 1$ supersymmetry requires $U^2 = U^3$ and allows for the two possibilities $\rho = 1, 1/3$. From (2.7) and (2.3) we obtain for the three gauge couplings ($\kappa_b = 2$) at the string scale:

$$\begin{aligned}\alpha_s^{-1} &= \frac{1}{2\sqrt{2}}\,\frac{M_{pl}}{M_s}\,\left(\frac{1}{\rho^2}\frac{1}{\sqrt{U^1}U^2} + 9\rho^2 \frac{U^2}{\sqrt{U^1}}\right), \\ \alpha_w^{-1} &= \frac{1}{2\sqrt{2}}\,\frac{M_{pl}}{M_s}\,\frac{\sqrt{U^1}}{2}, \\ \alpha_Y^{-1} &= \frac{1}{2\sqrt{2}}\,\frac{M_{pl}}{M_s}\,\left[\frac{2}{3}\left(\frac{1}{\rho^2}\frac{1}{\sqrt{U^1}U^2} + 9\rho^2 \frac{U^2}{\sqrt{U^1}}\right) + \frac{1}{2}\sqrt{U^1}\right].\end{aligned} \tag{2.8}$$

Immediatley, we see, that the condition (2.9) is satisfied.

even though it is less constraining. It is amusing that even though there seems to be no hidden SU(5) in our construction, we derived a compatible relation from intersecting branes. Note, that the relation for the internal volumes of the four D6-branes, $V_a = V_d$ and $V_b = V_c$, and in consequence the relation (2.9) might also hold for IBW models, even when there is no point in moduli space where the gauge symmetry is enhanced to $U(4) \times SU(2) \times SU(2)$. In fact, all the results we are going to derive in the following are true as long as (2.9) holds.

By making similar natural assumptions for the Standard Model realization of type B in (2.2), one can derive the same relation (2.9), where again at the bottom there lies an enhanced $U(4) \times U(2) \times U(2)$ Pati-Salam-like IBW model.

3. Running of the gauge couplings

Using the string prediction of the relation among the gauge couplings at the string scale, we can now use the one-loop running of the gauge couplings down to the weak scale. As a first step we are ignoring string threshold corrections, which for concrete models may be determined from the results derived in [40, 30].

In the absence of threshold corrections, the one-loop running of the three gauge couplings is described by the well known formulas

$$\begin{aligned} \frac{1}{\alpha_s(\mu)} &= \frac{1}{\alpha_s} + \frac{b_3}{2\pi} \ln\left(\frac{\mu}{M_s}\right) \\ \frac{\sin^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_w} + \frac{b_2}{2\pi} \ln\left(\frac{\mu}{M_s}\right) \\ \frac{\cos^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_Y} + \frac{b_1}{2\pi} \ln\left(\frac{\mu}{M_s}\right), \end{aligned} \tag{3.1}$$

where (b_3, b_2, b_1) are the one-loop beta-function coefficients for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$. Remember that for the matter spectrum of the MSSM these coefficients are given by $(b_3, b_2, b_1) = (3, -1, -11)$.[6] Using the relation (2.9) at the string scale yields

$$\frac{2}{3}\frac{1}{\alpha_s(\mu)} + \frac{2\sin^2\theta_w(\mu) - 1}{\alpha(\mu)} = \frac{B}{2\pi} \ln\left(\frac{\mu}{M_s}\right) \tag{3.2}$$

with

$$B = \frac{2}{3} b_3 + b_2 - b_1 \,. \tag{3.3}$$

Therefore, once a concrete string model is given, one can compute beta-function coefficients and use (3.2) to compute the unification scale by inserting the measured values of the couplings constants at the weak scale. In the following we use the following values for the Standard Model parameters taken from [41]

$$\begin{aligned} M_Z &= 91.1876(21)\,\text{GeV}\,, & \alpha_s(M_Z) &= 0.1172(20)\,, \\ \alpha(M_Z) &= \frac{1}{127.934(27)}\,, & \sin^2\theta_w(M_Z) &= 0.23113(15)\,. \end{aligned} \tag{3.4}$$

[6] Note, that the concrete model in [38] has additional non-chiral matter, in particular in the adjoint representations spoiling asymptotic freedom of $SU(3)_c$.

It is clear from (3.2) that the resulting value of the unification scale only depends on the combination B of the beta-function coefficients. Therefore, there exists a whole class of 3-generation supersymmetric intersecting brane world models with additional non-chiral matter that lead to the same prediction for the string scale. We will come back to this point in section 4.

Now, assuming the matter spectrum of the MSSM we get $B = 12$ and the resulting value for the unification scale turns out to be

$$M_X = 2.04 \cdot 10^{16}\,\mathrm{GeV}\,. \tag{3.5}$$

Since in (1.5) we still have the internal volume as an unfixed parameter, in contrast to the heterotic string, we can identify the string scale with the unification scale $M_s = M_X$.

Of course, for the individual gauge couplings at the string scale we get

$$\alpha_s(M_s) = \alpha_w(M_s) = \frac{5}{3}\alpha_Y(M_s) = 0.041\,, \tag{3.6}$$

which are just the supersymmetric GUT scale values with the Weinberg angle being $\sin^2\theta_w(M_s) = 3/8$. We conclude that the described class of realistic intersecting brane world models, under the assumption of a matter content with the same B coefficient as the MSSM matter content features perturbative "gauge coupling unification" at the GUT scale. Note, that the string prediction is weaker than the GUT prediction, as the latter leads to two conditions among the gauge couplings at the GUT scale, which allows to derive one non-trivial relation among the three couplings at the weak scale. In figure 2 we have depicted the running of the three gauge couplings, where for illustrative purposes we have shown the SU(5) coupling $\alpha' = \frac{5}{3}\,\alpha_Y$.

In order to match the values of the gauge coupling constants in eq. (3.6) with the string parameters (see eqs. (1.3) and (1.6)) one needs an internal Calabi-Yau space with volume being slightly larger that the inverse string scale M_s^{-1}. Concretely, for the interesting case, when the string scale M_s is chosen to be the GUT scale of $2 \cdot 10^{16}\,\mathrm{GeV}$ in accordance with (3.6), we derive the estimate

$$M_s^6\,\frac{V_6}{(2\pi)^6} = \frac{1}{8}\,g_{st}^2\,\frac{M_{pl}^2}{M_s^2} \sim 4.5 \cdot 10^4\,g_{st}^2 \tag{3.7}$$

on the volume V_6 in units of the string mass M_s. Therefore for a uniform Calabi-Yau space, with

$$V_6 = (2\pi)^6\,R^6\,, \tag{3.8}$$

this condition (3.7) boils down to:

$$M_s^6\,R^6 \sim 4.5 \cdot 10^4\,g_{st}^2\,. \tag{3.9}$$

Assuming $g_{st} = g_X$ this requirement is achieved for a uniform Calabi-Yau with radius R being of the size

$$M_s R = 5.32\,. \tag{3.10}$$

In addition, the complex structure moduli of the Calabi-Yau have to be tuned such that the individual couplings at the string scale match the values shown in eq. (3.6). Via eq. (1.6) this requires the following values for $V_a/\sqrt{V_6}$:

$$V_w = 2\ V_s = 0.235\ \sqrt{V_6}\,. \tag{3.11}$$

Setting $V_a = (2\pi)^3 R_a^3$ and assuming again $g_{st} = g_X$ this can be immediately translated to

$$M_s R_s = 2.6\,, \qquad M_s R_w = 3.3\,. \tag{3.12}$$

Note that in our discussion we have neglected the one-loop threshold corrections Δ_a. The latter can be shown to be indeed small for the $\mathcal{N} = 1$ sectors of the toroidal model of [38] by using the explicit one-loop threshold formula derived in [30].

Let us compare the intersecting brane world picture with the heterotic unification scenario. As emphasized already, at tree level the heterotic string scale is independent from the internal Calabi-Yau volume and is determined only in terms of the Planck mass and the string coupling constant with the effect that gauge coupling unification with the MSSM spectrum is not possible. One way to get consistent heterotic unification is to include the effect of one-loop gauge threshold corrections. Specifically it was shown in [10] that minimal string unification in heterotic orbifolds with MSSM spectrum in addition to moduli dependent threshold corrections is in principle possible provided that the radius of the orbifold space is enlarged compared to M_s^{-1}. So assuming a non-perturbative S-duality among the heterotic string and the brane world models, the splitting of the heterotic gauge couplings at M_s due the heterotic one-loop threshold corrections Δ_a is mapped to the various wrapped brane volumes V_a at string tree level in the brane world models. Furthermore, the smallness of the gauge couplings $\alpha_a(M_s)$ in heterotic string compactifications is related to the universal value of the heterotic string coupling g_{st} plus the size of the Δ_a; on the other hand in the dual intersecting brane world models, the small values for $\alpha_a(M_s)$ are essentially due to the small ratio M_s/M_{pl}.

4. Exotic matter

In general besides the chiral matter string theory contains also additional non-chiral matter. This is also localized on higher dimensional intersection loci of the $D6$ branes and also comes with multiplicity n_{ij} with $i,j \in \{a,b,c,d\}$. Here n_{ij} for $i \neq j$ counts the number of hypermultiplets in bifundamental respectively (anti-)symmetric representations of the gauge group, whereas for $i = j$ the number n_{ii} denotes the number of chiral multiplets in the adjoint representation of the gauge group. Adding such exotic matter to the MSSM matter changes of course the individual beta-function coefficients, but for the combination B one obtains

$$B = 12 - 2\,n_{aa} - 4\,n_{ab} + 2\,n_{a'c} + 2\,n_{a'd} - 2\,n_{bb} + 2\,n_{c'c} + 2\,n_{c'd} + 2\,n_{d'd} \tag{4.1}$$

which is always an even integer. Thus, even with exotic matter the beta-function combination B can only vary in steps of two. Note, that some sectors like n_{bc} or $n_{a'a}$ have dropped

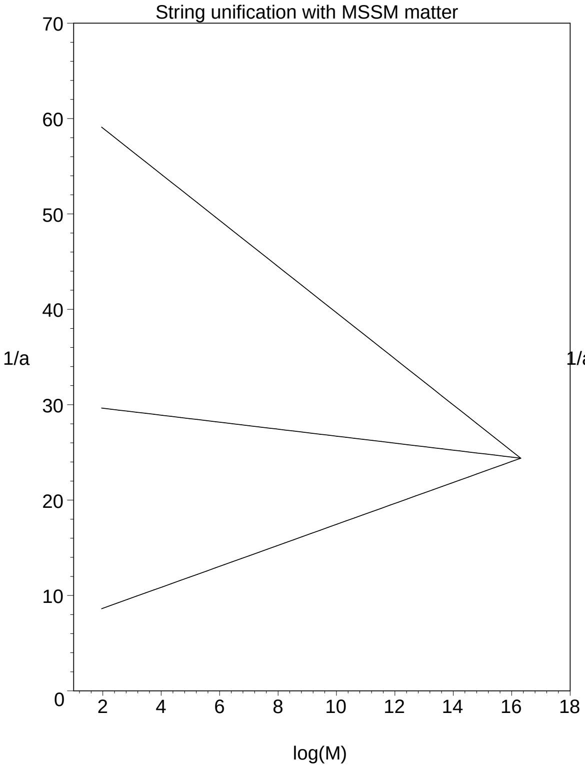

Figure 2: ??? .

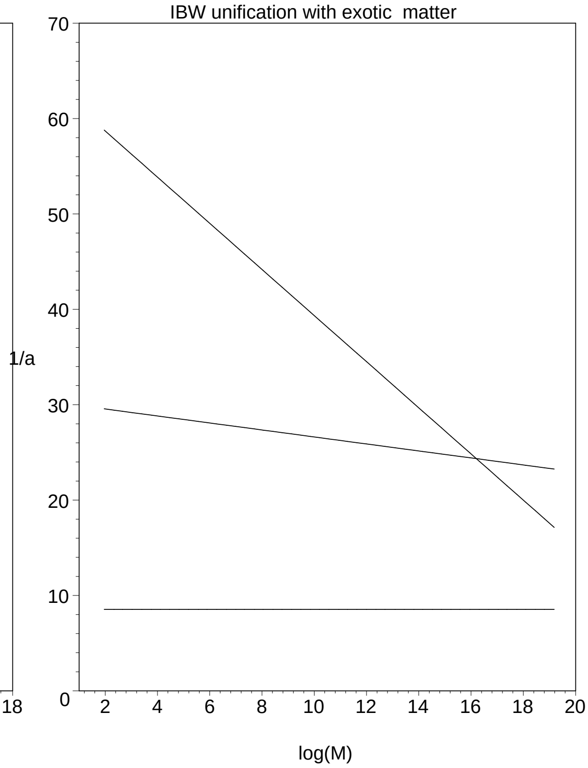

Figure 3: ??? .

B	18	16	14	12	10	8
$\mathbf{M_s}$[GeV]	$3.36 \cdot 10^{11}$	$5.28 \cdot 10^{12}$	$1.82 \cdot 10^{14}$	$2.04 \cdot 10^{16}$	$1.51 \cdot 10^{19}$	$3.06 \cdot 10^{23}$

Table 3: $M_s(B)$.

out completely and that some representations contribute positively and others negatively to B, so that cancellations are possible. In table 3, utilizing the relation (3.2), we show how the string scale depends on the beta-function parameter B

Interestingly for $B = 10$ there exists a value for M_s which is of the order of the Planck scale

$$\frac{M_s}{M_{pl}} = 1.24 \sim \sqrt{\frac{\pi}{2}}\,. \tag{4.2}$$

One example of this type is for instance given by choosing $n_{aa} = 1$, in which case the beta-function coefficients read $(b_3, b_2, b_1) = (0, -1, -11)$. The couplings at the string scale turn out to be

$$\alpha_s(M_s) = 0.117\,, \qquad \alpha_w(M_s) = 0.043\,, \qquad \alpha_Y(M_s) = 0.035 \tag{4.3}$$

leading to $\sin^2\theta_w(M_s) = 0.445$. As can be seen from figure 3, in this case not all three couplings intersect in one point, but nevertheless satisfy the relation (3.2) at the string scale. For the scales of the overall Calabi-Yau volume and the 3-cycles we obtain

$$M_s R = 0.6\,, \qquad M_s R_s = 1.9\,, \qquad M_s R_w = 3.3\,. \tag{4.4}$$

5. Conclusions

In this paper we have shown that under a few natural assumptions, realistic three generation supersymmetric intersecting brane world models lead to gauge coupling unification. We have argued that this result is quite robust against stringy contributions of additional exotic non-chiral matter. It is interesting to note that using the MSSM spectrum the relation (2.9) is accompanied by the accidental relation $\alpha_w = \alpha_s$. These relations are compatible with an SU(5) or SO(10) GUT, though emerge a priori without any reference to a simple gauge group. Therefore, one might wonder whether these relations find a group theoretical explanation by a (partial) gauge group unification which could happen in some decompactification limit where more branes are lying on top of each other.

So we like to emphasize again that the results presented in this article are due to a bottom up approach where we start from a few well motivated phenomenological assumptions. However to best of our knowledge so far there exists no explicit intersecting brane world construction which features all the necessary requirements, in particular provides the spectrum of the 3 generation MSSM without any exotic particles. In view of this, the challenge is even more pressing to construct realistic supersymmetric intersecting brane world models with a matter content satisfying $B \in \{10, 12\}$. With such a model one could compute more detailed informations about, for instance, whether the complex structure moduli can be chosen in such a way that eq (3.11) is fulfilled, and about the string threshold corrections citeTW,LustKY. We hope to report on this issue in the future [42].

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[42] Work in progress

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